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Subharmonic Structure in Superconducting Tunnel Junctions

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We discuss recent experiments which show that the intensity of the structure at $eV = 2\Delta/n$ (2Δ = energy gap, $n = 2, 3, 4, \ldots$) in a superconducting tunnel junction is proportional to the tunnel matrix element in the strong-coupling region. It is shown that for the odd series at $eV = 2\Delta/(2n+1)$ this behavior can be understood in terms of the accepted theory. However, the familiar explanation for the even series at $eV = 2\Delta/2n$ based on the sharp increase of the surface resistance of the superconducting films at $\hbar\omega = 2\Delta$ due to quasiparticle excitation is not consistent with the above experimental findings. A mechanism for the even series is proposed which involves the breaking of a tunneling Josephson pair caused by the oscillating electromagnetic field in the junction.

Recently, experiments have been reported which clearly identify the Josephson effect as the cause of the structure at $eV = 2\Delta/n$ ($n = 2, 3, 4, \cdots$) that is sometimes apparent in superconducting tunnel junctions.[1] By using junctions with a light-sensitive barrier it was possible to study the dependence of the structure on the tunnel matrix element $|M|^2$.

Typically, junctions which had no structure in the original dark state showed a rapidly increasing subharmonic structure as $|M|^2$ was increased. Upon further increase of $|M|^2$ the intensity of the structure became proportional to the rest of the current, i.e., to $|M|^2$. In some cases this proportionality could be verified over three decades of variation in $|M|^2$ for the $n = 2, 3, 4$ members of the subharmonic series. While it is obvious that this proportionality positively excludes multiparticle tunneling[2] as the cause of the structure, it is not easy to predict such a dependence from the Josephson equations of the junction.

The purpose of this communication is to discuss whether this proportionality can be explained in terms of present-day theories and models of the subharmonic structure, or whether new models have to be introduced. The discussion is based on the work of Werthamer[3] who has given a formal treatment of the mechanisms leading to subharmonic structure.

Werthamer develops the general expression for the total tunneling current of a junction in the presence of an arbitrary electromagnetic field. As can be seen from Werthamer's equation (11),[3] the dc component of the current shows structure at $eV = 2\Delta/(2n+1)$. Two processes are responsible for the structure. The first corresponds to a quasiparticle tunneling induced by the Josephson radiation analogous to the Dayem-Martin[4] process. The second is based on the singularity of the pair current amplitude predicted by Riedel[5] which should occur if the applied voltage plus a photon energy equals the energy gap. Both processes predict a proportionality to $|M|^2$ of the intensity of the structure only if the radiation amplitude in the junction is independent of $|M|^2$ [Eq. (36) in Ref. 3]. For $eV^{AC}/\hbar\omega \gg 1$ (strong-coupling region) the intensity depends rather weakly on the radiation amplitude, and a qualitative saturation-type behavior of the radiation in the strong-coupling region is sufficient to be consistent with the experiment.

The familiar explanation of the even series is based on the sharp increase of the surface resistance of a superconductor at $\hbar\omega = 2\Delta$ due to quasiparticle excitation. The power absorbed by this process is proportional to the radiation density or the square of the current amplitude. To be consistent with the experiment the ac Josephson field would have to be proportional to $(|M|^2)^{1/2}$ in the strong-coupling region which contradicts the saturation-type behavior required to explain the odd series.

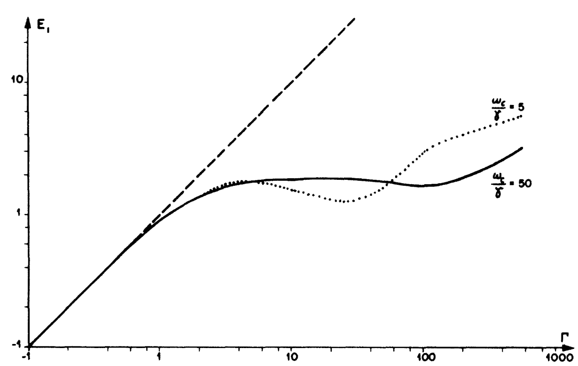

FIG. 1. Amplitude E (in arbitrary units) of the ac Josephson field as a function of the coupling constant $\Gamma = \omega_J^2/\omega_c$ for two values of the damping constant γ. Γ is proportional to $|M|^2$. The calculation has been performed for resonance condition $\omega = \omega_c$ and the fundamental frequency component of the field is plotted. Around $\Gamma = 1000$ the computer solution of Eq. (1) became unstable.

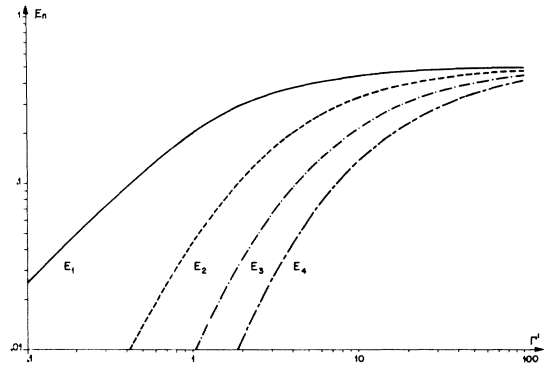

FIG. 2. Amplitudes $|E_n|$ of the Fourier components with frequency $2neV_{dc}/\hbar$ of the ac Josephson field in arbitrary units for $n=1,2,3,4$ as a function of the coupling constant $\Gamma' = (\hbar\omega_J/2eV_{dc})^2$ according to Eqs. (3) and (4).

In order to decide between the two possibilities, we have calculated the ac Josephson amplitude as a function of $|M|^2$. Werthamer has shown that subject to certain conditions the Josephson equations of a tunnel junction become equivalent to

$$\left(\frac{d^2}{dt^2}+\gamma\frac{d}{dt}+\omega_c^2\right)\Omega(t)=\omega_J^2\frac{d}{dt}\cos\Phi(t), \qquad \frac{d\Phi(t)}{dt}=\Omega(t)+\Omega_{dc}, \tag{1}$$

where ω_c and γ are an effective cavity-resonance frequency and a damping constant, respectively. The Josephson plasma frequency ω_J is defined in Ref. 3 and measures the coupling, i.e., $\omega_J^2 \approx |M|^2$, and $\Omega_{dc}=2eV/\hbar$ corresponds to the applied voltage V. Since we are interested in the strong-coupling region $\Gamma=\omega_J^2/\omega_c\gamma \gg 1$, we cannot use Werthamer's approximate analytical solution which is good in the weak- and intermediate-coupling region. In Fig. 1, a computer calculation of Eq. (1) is shown. Plotted is the amplitude of the ac Josephson field at fundamental frequency $\omega = 2eV/\hbar$ as a function of $\Gamma=\omega_J^2/\omega_c\gamma \propto |M|^2$ for two different damping constants and at $\omega=\omega_c$.

For the case when stripline and cavity-resonance effects are unimportant and the damping is weak ($\omega_c=\gamma=0$), Eq. (1) reduces to the simple pendulum equation

$$\ddot{\phi}+\omega_J^2\sin\phi=0. \tag{2}$$

We obtain the following parameter representation for the Fourier components E_n at frequency $\hbar\omega = 2neV_{dc}$ of the oscillating electromagnetic field:

$$E_n=(-1)^n\left[2\cosh\left(n\pi\frac{K(1-m)}{K(m)}\right)\right]^{-1}, \tag{3}$$

$$\Gamma'=\frac{\hbar\omega_J}{2eV_{dc}}=K^2(m)\frac{m}{\pi^2}, \tag{4}$$

where K is the complete elliptic integral of the first kind. Equations (3) and (4) can be expanded, and we obtain

$$E_n=(-\tfrac{1}{4}\Gamma')^n \ll 1, \tag{5}$$

$$E_n=\frac{(-1)^n}{2\cosh(\frac{1}{2}n\pi\Gamma'^{-1/2})}+O(e^{-4\Gamma'}), \quad \Gamma'>1. \tag{6}$$

For $\Gamma'\to\infty$ all $|E_n|\to\frac{1}{2}$ as can be seen from Eq. (6). In Fig. 2 $|E_1|$ to $|E_4|$ are plotted as functions of the coupling constant Γ'.

As can be seen from Figs. 1 and 2, the amplitudes show a saturation-type behavior and thus make Werthamer's model for the odd series consistent with the experiment. However, it clearly rules out an absorption-coefficient-type mechanism, such as the frequency-dependent surface resistance, as the cause for the even series.

We are therefore left with the problem of finding a mechanism for the even series which gives the correct $|M|^2$ dependence. A possible generalization of Werthamer's equations would be to consider not only the action of the ac fields on the phase, but also to include pair-breaking effects.

For instance, it is energetically possible for a tunneling Josephson pair at $eV=2\Delta/2$ to break up giving a structure in the dc current at that voltage. Equation (9.13c) in Ref. 6 shows, however, that the current has only a dc component if the

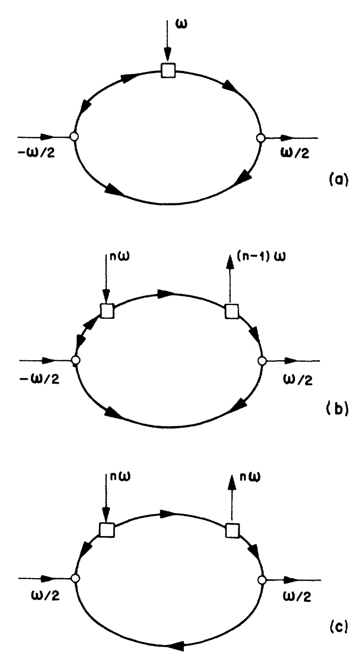

FIG. 3. Examples of diagrams that contribute to the midgap structure, (a), the higher terms in the even series (b), and the odd series (c). The squares stand for the interaction with the oscillating electric field generated by the Josephson current.

pair-breaking interaction has the correct time dependence $\omega = 2eV/\hbar$. The ac Josephson field of the junction has this time dependence.

Figure 3(a) shows a diagrammatic expansion of the tunneling current using the standard notation of Ref. 6. The proposed mechanism consists of the breaking of a pair under the influence of an oscillating field and leads to structure at $eV = 2\Delta/2$ ($\Delta = \Delta_1, \Delta_2$, respectively).

The higher-order members of the even series at $2\Delta/2n$ (i.e., Δ_1/n and Δ_2/n with $n = 2, 3, \cdots$) require the absorption of a Josephson photon. Figure 3(b) shows the corresponding diagram which includes the interaction with the oscillating field only to lowest order. Both Figs. 3(a) and 3(b) give structure proportional to $|M|^2$ if the ac Josephson amplitudes saturate.

While the diagrams 3(a) and 3(b) for the even series give structure in the pair current [(9.13c) in Ref. 6], Fig. 3(c), which is given for completeness only, produces structure in the single-particle current [(9.13b) in Ref. 6] at $eV = 2\Delta/(2n+1)$, as discussed by Werthamer.

Figures 3(b) and 3(c) are similar and involve two interactions with the oscillating field, while 3(a) involves only one interaction. This is consistent with the experimental observation that the mid-gap structure shows up before the rest of the structure and that the higher-order even series and the odd series are of comparable intensity.

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Spin Correlation and Entropy, H. Falk and Masuo Suzuki [Phys. Rev. B 1, 3051 (1970)]. Several typographical errors (which do not propagate to the results) should be noted: In each of Eqs. (1.5)–(1.7) the extra ln terms should be deleted. In Eq. (3.11) the sine and cosine definitions should be interchanged. The left-hand side of Eq. (3.24) should read $2^{-N}Z$.